Non-Thermal and Dust Charge Effects on Surface Ion Waves in Semi-Bound Lorentzian Plasmas

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The non-thermal and dust charge effects on a surface electrostatic ion plasma wave are investigated in a semi-bound magnetized dusty Lorentzian plasma. The results show that the phase velocity of the surface wave with negatively charged dust grains is greater than that with positively charged dust grains or that with neutral dust grains. It is also found that the phase velocity increases with increasing the spectral index of the plasma. For the long wavelength domain, however, the phase velocity of the surface wave is found to be almost independent of the spectral index.

Key words: Surface Ion Waves; Lorentzian Plasmas.

1. Introduction

Various surface plasma waves [1 – 7] propagating along the plasma-vacuum interface have attracted much attention since the frequency spectra have wide applications in many areas such as laser physics, plasma technology, and plasma spectroscopy. Recently, there has been a considerable interest in the dynamics of plasmas containing dust grains including collective effects and strong electrostatic interactions. The various physical processes in dusty plasmas have been investigated in order to obtain information on the plasma parameters in dusty plasmas [8, 9]. It has been found that the velocity distributions in dusty plasmas usually do not resemble the isotropic Maxwellian velocity distributions but have more complicated shapes exhibiting high-energy tails which strongly deviate from simple Maxwellians due to the temperature anisotropy caused by the coupling between plasmas and external fields. In recent years, the Lorentzian distribution [10 – 12] has been of great interest since it is known that the Lorentzian distribution is quite useful for investigating physical properties of magnetized dusty plasmas. It has been also found that the effect of dust charge plays an important role in propagating plasma waves in dusty plasmas [13, 14]. However, the dust grain and non-thermal effects on the dispersion properties of the surface electrostatic ion plasmas wave in a semi-bound magnetized dusty Lorentzian plasma have not been investigated yet. Thus, in this paper we investigate the dust charge and non-Maxwellian effects on the surface electrostatic ion plasma wave in a semi-bound magnetized dusty Lorentzian plasma. The investigation of the dispersion properties of the surface wave in magnetized dusty Lorentzian plasmas would be a useful tool for investigating the physical properties of bound or semi-bound dusty plasmas. Here, we consider the propagation of the surface ion plasma wave along the plasma-vacuum interface. The specular reflection condition [1 – 4] is known to be particularly useful to investigate the dispersion properties of surface waves propagating along the plasma interface in semi-bound isotropic plasmas. The magnetic field effect could introduce anisotropy in the distribution. However, when the static magnetic field is parallel to the surface boundary, the specular reflection condition would be reliable to describe the surface plasma waves propagating along the plasma-vacuum interface.

In Section 2, we obtain the dispersion relation of a surface ion plasma wave in semi-bound magnetized dusty Lorentzian plasmas using the specular reflection condition. In Section 3, we investigate the variation of the phase velocity of a surface ion plasma wave propagating along the plasma-vacuum interface due to the change of the spectral index and the sign of the dust grains. The conclusions are given in Section 4.

2. Dispersion Relation

The specular reflection condition [1, 2] for surface electromagnetic waves propagating along the z-direction in a semi-bound plasma with the plasma-vacuum interface at $x = 0$ expressed as

$$\left(\frac{k_\parallel^2 c^2}{\omega^2} - 1\right)^{1/2} + \frac{\omega}{\pi c}\int_{-\infty}^{\infty} \frac{\mathrm{d}k_\perp}{k^2}\left[\frac{k_\parallel^2 c^2}{\omega^2 \varepsilon_\mathrm{l}(\omega,k)} - \frac{k_\perp^2 c^2}{k^2c^2 - \omega^2\varepsilon_\mathrm{t}(\omega,k)}\right] = 0, \quad (1)$$

where $k^2 = k_\perp^2 + k_\parallel^2$, $k_\perp\,(= k_x)$ and $k_\parallel\,(= k_z)$ are, respectively, the perpendicular and parallel components of the wave vector $\boldsymbol{k}$; ω is the frequency, c the speed of the light and $\varepsilon_\mathrm{l}(\omega,k)$ and $\varepsilon_\mathrm{t}(\omega,k)$ are the longitudinal and transverse components of the plasma dielectric function, respectively. In this geometry, the y-coordinate can be ignored without loss of generality since the y-coordinate is a translational invariance. This condition of mirror reflection has commonly been used for the investigation of the surface wave propagation in homogeneous plasmas [2]. Equation (1) is also called the electromagnetic surface wave dispersion relation [3]. It should be also noted that (1) is valid strictly in a plasma with isotropic velocity distribution [4]. It is well known that the physical properties of various electrostatic waves in plasmas can be obtained by the plasma dielectric function. In magnetized dusty Lorentzian plasmas, the longitudinal component of the plasma dielectric function can be expressed by the plasma dielectric susceptibilities

$$\varepsilon_\mathrm{l}(\omega,k) = 1 + \chi_\mathrm{e} + \chi_\mathrm{i} + \chi_\mathrm{d}, \quad (2)$$

where χ_α are the dielectric susceptibilities [12] for electrons ($\alpha = \mathrm{e}$), ions ($\alpha = \mathrm{i}$), and dust grains ($\alpha = \mathrm{d}$):

$$\chi_\alpha = \frac{1}{k^2\lambda_{\kappa\alpha}^2}\left[1 + \frac{\mathrm{i}\omega}{2^{\kappa-1/2}\Gamma(\kappa+1/2)}\int_0^\infty \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau} z_\alpha^{\kappa+1/2}(\tau) K_{\kappa+1/2}(z)\right]. \quad (3)$$

Here, the parameter function $z_\alpha(\tau)$ [10] is given by

$$z_\alpha(\tau) = (2\kappa)^{1/2}\left[\frac{k_\perp^2\theta_\alpha^2}{\omega_{\mathrm{c}\alpha}^2}(1-\cos\omega_{\mathrm{c}\alpha}\tau) + \frac{1}{2}k_\parallel^2\theta_\alpha^2\tau^2\right]^{1/2}. \quad (4)$$

$\lambda_{\kappa\alpha}$ $[\equiv ((2\kappa-3)/(2\kappa-1))^{1/2}\lambda_{\mathrm{D}\alpha}]$ is the appropriate Debye length for a plasma with the Lorentzian velocity distribution, κ is the spectral index, $\lambda_{\mathrm{D}\alpha}$ the usual Debye length of the particle of species α, Γ the gamma function, K_n the modified Bessel function [15] with order n, $\omega_{\mathrm{c}\alpha}$ the cyclotron frequency, $\theta_\alpha = [(2\kappa-3)/\kappa]^{1/2}(T_\alpha/m_\alpha)^{1/2}$, T_α and m_α are the temperature and mass of the species α. When an external magnetic field is parallel to the surface boundary, for low-frequency modes in magnetized dusty Lorentzian plasmas, i. e., $\omega_\mathrm{cd}, kv_{T\mathrm{i}}, kv_{T\mathrm{d}} \ll \omega \ll k_z v_{T\mathrm{e}}, \omega_\mathrm{ce}k_z/k_\perp$, $kv_{T\mathrm{e}} \ll \omega_\mathrm{ce}$, $kv_{T\mathrm{i}} \ll \omega_\mathrm{ci}$, and $Z_\mathrm{d} \ll m_\mathrm{d}/m_\mathrm{i}$, where Z_d is the charge number of the dust grain, the plasma dielectric function would be found as

$$\varepsilon_\mathrm{l}(\omega,k_\perp,k_\parallel) \cong 1 + \frac{1}{\mu_\kappa(k_\perp^2+k_\parallel^2)\lambda_\mathrm{De}^2} - \frac{\omega_\mathrm{pi}^2 k_\perp^2}{(\omega^2-\omega_\mathrm{ci}^2)(k_\perp^2+k_\parallel^2)} - \frac{\omega_\mathrm{pi}^2 k_\parallel^2}{\omega^2(k_\perp^2+k_\parallel^2)} - \frac{\omega_\mathrm{pd}^2}{\omega^2}, \quad (5)$$

where $\mu_\kappa \equiv (2\kappa-3)/(2\kappa-1)$ and $\omega_{\mathrm{p}\alpha}$ is the plasma frequency. The specular reflection condition in the quasi-static limit, i. e., $\omega^2\varepsilon/c^2 \ll k^2$, is then given by

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_\perp k_\parallel}{k^2\varepsilon_\mathrm{l}(\omega,k_\perp,k_\parallel)} = -1. \quad (6)$$

Equation (6) is also called the electrostatic surface wave dispersion relation [3]. Hence, the dispersion relation of the low-frequency electrostatic surface ion plasma wave in a semi-bound magnetized dusty Lorentzian plasma is obtained by the contour integration

$$\oint \mathrm{d}k_\perp k_\parallel \left[\left(1-\frac{\omega_\mathrm{pd}^2}{\omega^2}\right)(k_\perp^2+k_\parallel^2) - \frac{\omega_\mathrm{pi}^2 k_\perp^2}{\omega^2-\omega_\mathrm{ci}^2} - \frac{\omega_\mathrm{pi}^2 k_\parallel^2}{\omega^2} + \frac{1}{\mu_\kappa\lambda_\mathrm{De}^2}\right]^{-1} + \pi = 0. \quad (7)$$

3. Phase Velocity of a Surface Ion Wave

Using the residue analysis at the poles in the $k_\perp$-plane, we obtain the following expression of the dispersion relation:

$$\frac{\omega^2(\omega^2-\omega_{\rm ci}^2)k_\parallel^2\lambda_{\rm De}^2}{\omega^2(\omega^2-\omega_{\rm ci}^2)-\omega_{\rm pd}^2(\omega^2-\omega_{\rm ci}^2)-\omega^2\omega_{\rm pi}^2} - \frac{[\omega^2-(\omega_{\rm ci}^2+\omega_{\rm pd}^2)]k_\parallel^2\lambda_{\rm De}^2}{\omega^2} - \frac{1}{\mu_\kappa} = 0. \quad (8)$$

For low-frequency cases, i. e., $\omega \ll \omega_{\rm ci}$, the dispersion relation for the electrostatic surface ion plasma wave in a semi-bound magnetized dusty Lorentzian plasma is then found to be

$$\omega^4\left[\mu_\kappa^{-1}\lambda_{\rm De}^{-2}(\omega_{\rm ci}^2+\omega_{\rm pi}^2)+k_\parallel^2\omega_{\rm pi}^2\right] - \omega^2\left[(\mu_\kappa^{-1}\lambda_{\rm De}^{-2}+k_\parallel^2)\omega_{\rm pd}^2\omega_{\rm ci}^2+k_\parallel^2(\omega_{\rm pi}^2+\omega_{\rm pd}^2)(\omega_{\rm ci}^2\omega_{\rm pi}^2)\right] + k_\parallel^2(\omega_{\rm pi}^2+\omega_{\rm pd}^2)\omega_{\rm pd}^2\omega_{\rm ci}^2 = 0. \quad (9)$$

The charge neutrality condition in dusty plasmas is represented by the relation $Z_{\rm d}n_{\rm d}+\delta-1=0$, where $\delta \equiv n_{\rm i}/n_{\rm e}$ and n_α is the density of the species α. Hence, $\delta<1$ for positively charged dust grains, i. e., $Z_{\rm d}>0$, and $\delta>1$ for negatively charged dust grains, i. e., $Z_{\rm d}<0$. Here we observe that the sign of the charged dust grains in dusty plasmas can be characterized by the parameter δ. Then, $k_\parallel\lambda_{\kappa\rm e}$ can be expressed by $k_\parallel\lambda_{\kappa\rm e}=\delta(T_{\rm e}n_{\rm e}/T_{\rm i}n_{\rm i})^{1/2}[(2\kappa-3)/(2\kappa-1)]^{1/2}\tilde{k}_\parallel$, where $\tilde{k}_\parallel\,(\equiv k_\parallel\lambda_{\rm Di})$ is the scaled wave length. After some manipulations, the low-frequency mode solution of the dispersion relation for an electrostatic surface ion plasma wave in a semi-bounded magnetized dusty Lorentzian plasma is found to be

$$\begin{aligned}
\frac{\omega(\tilde{k}_\parallel,\kappa,T_{\rm e}/T,\delta)}{\omega_{\rm ci}} &= \left\{2\left[1+\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2+\frac{T_{\rm e}}{T_{\rm i}}\frac{2\kappa-3}{2\kappa-1}\delta\tilde{k}_\parallel^2\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2\right]\right\}^{-1/2}\\
&\cdot\left\{\left[\left[1+\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2+\frac{T_{\rm e}}{T_{\rm i}}\frac{2\kappa-3}{2\kappa-1}\delta\tilde{k}_\parallel^2\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2\right]\left(\frac{\omega_{\rm pd}}{\omega_{\rm ci}}\right)^2+\frac{T_{\rm e}}{T_{\rm i}}\frac{2\kappa-3}{2\kappa-1}\delta\tilde{k}_\parallel^2\left[\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2+\left(\frac{\omega_{\rm pd}}{\omega_{\rm ci}}\right)^2\right]\left[1+\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2\right]\right]\right.\\
&-\left[\left[1+\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2+\frac{T_{\rm e}}{T_{\rm i}}\frac{2\kappa-3}{2\kappa-1}\delta\tilde{k}_\parallel^2\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2\right]\left(\frac{\omega_{\rm pd}}{\omega_{\rm ci}}\right)^2+\frac{T_{\rm e}}{T_{\rm i}}\frac{2\kappa-3}{2\kappa-1}\delta\tilde{k}_\parallel^2\left[\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2+\left(\frac{\omega_{\rm pd}}{\omega_{\rm ci}}\right)^2\right]\left[1+\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2\right]\right]^2\\
&\left.-4\left[\left[1+\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2+\frac{T_{\rm e}}{T_{\rm i}}\frac{2\kappa-3}{2\kappa-1}\delta\tilde{k}_\parallel^2\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2\right]\frac{T_{\rm e}}{T_{\rm i}}\frac{2\kappa-3}{2\kappa-1}\delta\tilde{k}_\parallel^2\left[\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2+\left(\frac{\omega_{\rm pd}}{\omega_{\rm ci}}\right)^2\right]\left(\frac{\omega_{\rm pi}}{\omega_{\rm ci}}\right)^2\right]^{1/2}\right\}.
\end{aligned} \quad (10)$$

In order to investigate the non-thermal and dust grain effects on the phase velocity of the surface wave in a dusty plasma, we choose $\omega_{\rm pi}/\omega_{\rm ci}=300$, $\omega_{\rm pd}/\omega_{\rm pi}=10^{-5}$, and $\omega_{\rm pd}/\omega_{\rm ci}=10^{-3}$. Figures 1 and 2 show the dispersion relations as functions of the scaled wave number $\tilde{k}_\parallel$ for various values of the parameter δ for small and large spectral indices, respectively. Figure 3 represents the three-dimensional plot of the dispersion relation as a function of the parameter δ and the scaled wave number $\tilde{k}_\parallel$. As we see in these figures, it is found that the phase velocity of the surface ion plasma wave in dusty plasmas with negatively charged dust grains is found to be greater than that with positively charged dust grains or that with neutral dust grains. It is understood that the ion density will be greater than the electron density if the dust grain is negatively charged according to the charge neutrality condition. Hence, the increase of the ratio of the ion density to the electron density causes the increase of the phase velocity of the surface ion wave in dusty plasmas. In addition, it is found that the phase velocity is closely related to the non-thermal character of the semi-bound plasma, since the phase velocity increases with increasing spectral index. Thus, it should be noted that the phase velocity of the surface ion wave in the thermal plasma is always greater than that in the non-thermal plasma. Figures 4 and 5 show the three-dimensional plots of the dispersion relation as functions of the spectral index κ and the scaled wave number $\tilde{k}_\parallel$ for negatively and positively charged dust grains, respectively. It is found that the phase velocity of the surface wave in long wavelength regions is

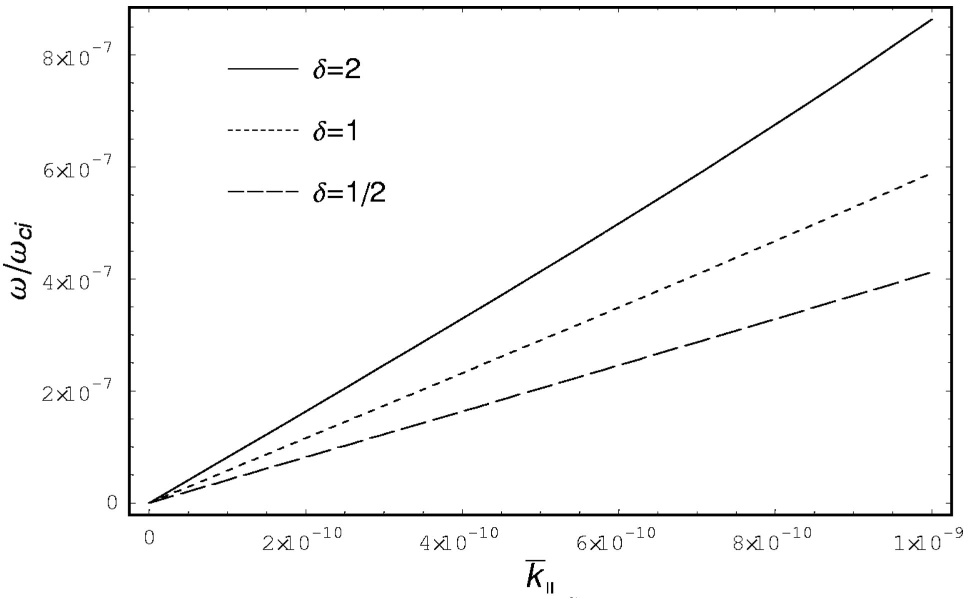

Fig. 1. The dispersion relation as a function of the scaled wave number $\tilde{k}_{\parallel}$ for $\kappa = 2$ and $T_e = T_i$. The solid line represents the case of $\delta = 2$, i. e., negatively charged dust grains. The dotted line represents the case of $\delta = 1$, i. e., neutral dust grains. The dashed line represents the case of $\delta = 1/2$, i. e., positively charged dust grains.

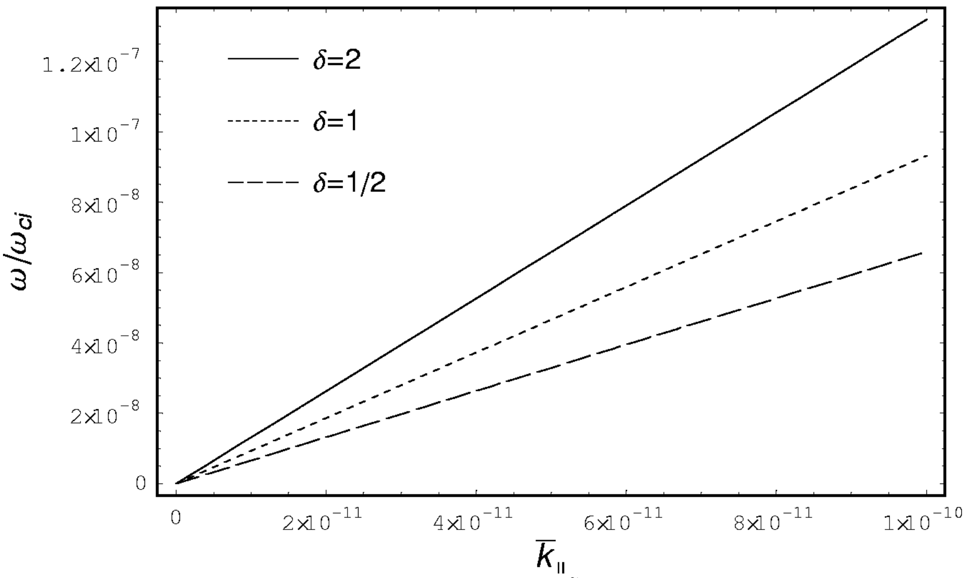

Fig. 2. The dispersion relation as a function of the scaled wave number $\tilde{k}_{\parallel}$ for $\kappa = 8$ and $T_e = T_i$. The conditions are the same as in Figure 1.

almost independent of the spectral index κ, since all terms including the spectral index become negligible when the scaled wave number is very small. Therefore it should be noted that the surface ion wave is almost independent of the non-thermal effect in long wavelength domains. However, we also observe that the non-thermal character of the magnetized dusty Lorentzian plasma suppresses the phase velocity of the surface ion plasma wave with increasing the wave number.

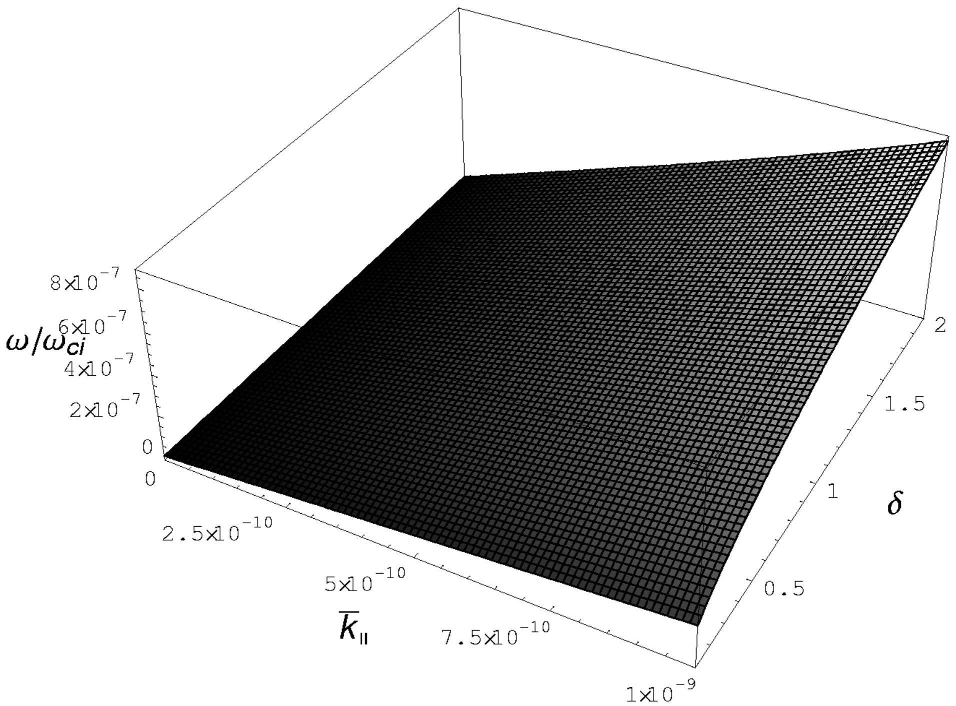

Fig. 3. Three-dimensional plot of the dispersion relation as a function of the parameter δ and the scaled wave number $\tilde{k}_{\|}$ for $\kappa = 2$ and $T_e = T_i$.

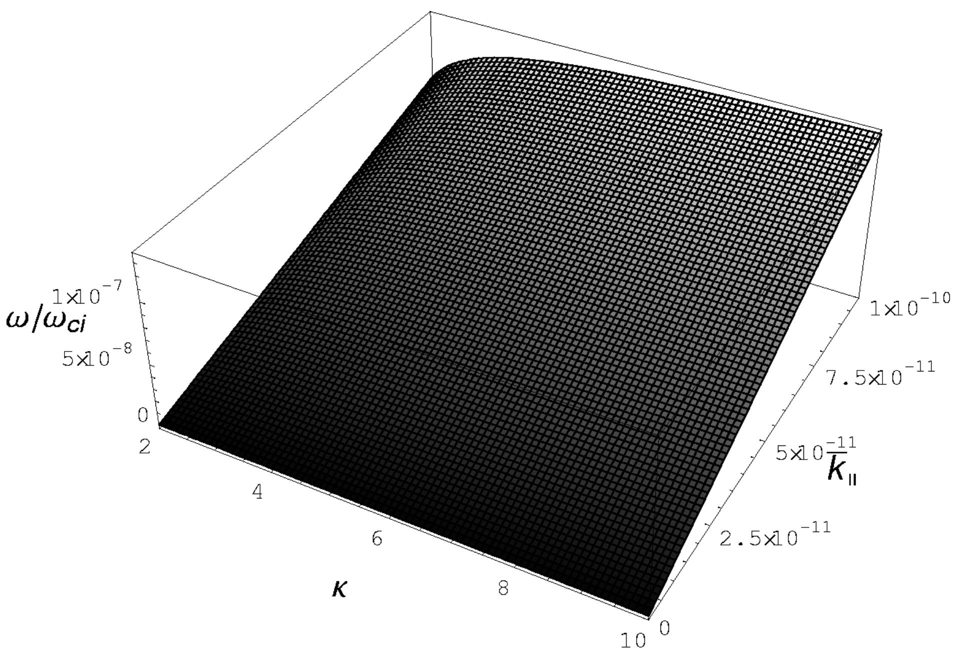

Fig. 4. Three-dimensional plot of the dispersion relation as a function of the spectral index κ and the scaled wave number $\tilde{k}_{\|}$ for $T_e = T_i$ and $\delta = 2$, i. e., negatively charged dust grains.

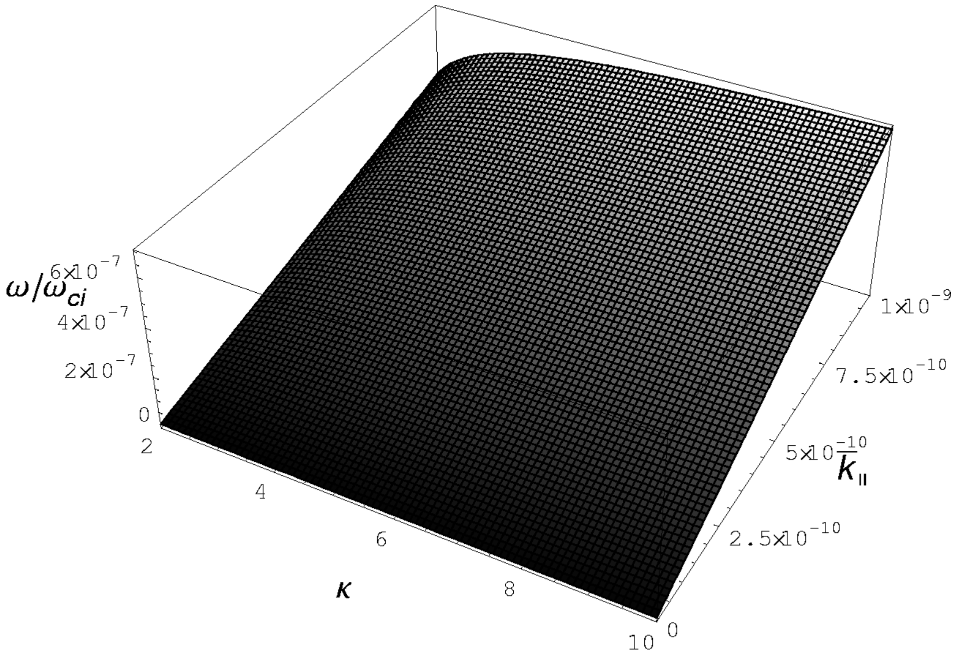

Fig. 5. Three-dimensional plot of the dispersion relation as a function of the spectral index κ and the scaled wave number $\tilde{k}_{\parallel}$ for $T_e = T_i$ and $\delta = 1/2$, i. e., positively charged dust grains.

4. Conclusions

We investigate the non-thermal and dust grain effects on the surface electrostatic ion plasma wave in a semi-bound magnetized dusty Lorentzian plasma. The specular reflection boundary condition is applied to obtain the dispersion relation of the surface ion plasma wave, using the plasma dielectric function. It is found that the phase velocity of the surface wave with negatively charged dust grains is greater than that with positively charged dust grains or that with neutral dust grains. In addition, it is also found that the phase velocity increases with increasing the spectral index of the magnetized dusty Lorentzian plasma. Thus, it is understood that the phase velocity of the surface ion wave in the thermal plasma is always greater than that in the non-thermal plasma. However, for long wavelength domains, the phase velocity of the surface wave is found to be almost independent of the spectral index. It is also found that the non-Maxwellian character of the magnetized dusty Lorentzian plasma suppresses the phase velocity of the surface ion plasma wave with increasing the wave number. These results are important for understanding the effects of dust grains and the non-thermal character of the plasma on low-frequency surface ion plasma waves in a semi-bound magnetized dusty Lorentzian plasma.

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